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Suppressing chaos in lasers by negative feedback

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An easy to implement method for stabilizing periodic orbits in a modulated laser is presented. Such a method is based on negative feedback of subharmonic components extracted from the temporal intensity signal. Robustness, speed, and general validity of this scheme for other laser systems are also discussed. [S1063-651X(96)50906-2]

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In the last few years different schemes for controlling chaos to periodic orbits have been proposed. The main common characteristic of all these schemes is that the chaotic motion in the phase space can be directed into a required periodic orbit by applying tiny perturbations. In this way, a relevant change in the dynamical behavior can be induced while the energy required for the control should be as small as possible.

In order to perform a simple classification of control methods, two main categories can be defined: feedback and nonfeedback methods. The main feedback techniques, which allow stabilization of orbits embedded in the chaotic attractor, are the Ott-Grebogi-Yorke (OGY) method [1] (with its implementations known as occasional proportional feedback (OPF) [2] and minimal expected deviation (MED) [3]), and the self controlling feedback methods [4,5]. The OGY, OPF, and MED methods consist in adjusting an accessible control parameter each time the system passes through a given Poincaré section to guide the trajectory to a selected orbit (corresponding to a fixed point in the Poincaré section). The OGY algorithm presents some difficulties if applied to fast dynamics, since the state of the systems must be accurately monitored and the feedback signal suddenly changed when the trajectory crosses the Poincaré section. On the contrary, the self controlled feedback schemes use a continuous rather than abruptly changed feedback signal of the form $\varepsilon(t) \sim \gamma[x(t)-x(t-\tau)]$, where $x(t)$ is a dynamical variable and τ is the period of the desired periodic orbit. Implementations [5] of the original scheme proposed by Pyragas are based on the introduction of a suitable weight factor on the delayed variable $x(t-\tau)$ or on variability of the gain γ. In this latter case, the strength of the perturbation is driven by the local information extracted from the dynamics itself.

Nonfeedback methods deal with the application of small driving forces [6,7]. These methods slightly modify the dynamics of the system such that stable solutions appear. The main advantage of nonfeedback schemes lies in their speed; indeed, no on-line monitoring and processing is required. Actually, they have been successfully applied in different experimental frames. In addition to the above methods based on small perturbations, there exist model-based approaches of open loop control, introduced in Ref. [8].

In this paper we will show that the chaotic behavior of a CO_2 laser with modulated losses can be controlled by a negative feedback of a suitable spectral component of the laser intensity. This control scheme (which can be obviously classified as a feedback scheme) presents the relevant advantage of being robust and fast, together with the characteristic of requiring very low energy.

First of all, it is useful to recall some preliminary results on the CO_2 laser dynamics. The behavior of the CO_2 laser with modulated losses is quantitatively matched by a four level scheme, which, besides the relatively fast radiative coupling between the resonant molecular transition (populations N_1, N_2) and the field intensity (I), accounts also for the relatively slower collisional transfer from the manifold of the other rotational levels (populations M_1, M_2). Thus, the two energy flows imply a set of five differential equations as follows:

$$\dot{I} = -\kappa_0[1 + m\sin(2\pi f t)]I + G(N_2 - N_1)I,$$

$$\dot{N}_2 = -(z\gamma_R + \gamma_2)N_2 - G(N_2 - N_1)I + \gamma_R M_2 + \gamma_2 P,$$

$$\dot{N}_1 = -(z\gamma_R + \gamma_1)N_1 + G(N_2 - N_1)I + \gamma_R M_1,$$

$$\dot{M}_2 = -(\gamma_R + \gamma_2)M_2 + z\gamma_R N_2 + z\gamma_2 P,$$

$$\dot{M}_1 = -(\gamma_R + \gamma_1)M_1 + z\gamma_R N_1, \tag{1}$$

where $\kappa_0 = 3.18\times 10^7\ \mathrm{s}^{-1}$ is the intensity decay-rate, $\gamma_R = 7.0\times 10^5\ \mathrm{s}^{-1}$ is the relaxation rate between the lasing states and the associated rotational manifolds (the enhancement factor $z = 10$ represents the number of sublevels considered in each manifold), $\gamma_1 = 8.0\times 10^4\ \mathrm{s}^{-1}$ and $\gamma_2 = 1.0\times 10^4\ \mathrm{s}^{-1}$ are the relaxation rates of the vibrational states, $G = 8.75\times 10^{-8}\ \mathrm{s}^{-1}$ is the field-matter coupling constant, and the dimensional parameter $P = 5.46\times 10^{14}$ represents the pump (the numerical values of these quantities are chosen following Ref. [9]). The parameters m and f represent the amplitude and the frequency of the external driving signal. It is well known that fixing f in the range [70 kHz, 140 kHz] and increasing the control parameter m, the system undergoes a subharmonic cascade (with fundamental period $T = 1/f$) ending in chaos.

For a wide working range, this five dimensional model can be reduced to the following set of three differential equations [10]:

$$\dot{x}_1 = \kappa'[x_2 - (1 + m\sin\Omega\gamma_R\tau)],$$

$$\dot{x}_2 = -\Gamma_1 x_2 - 2\kappa' x_2 e^{x_1} + P_0 + P_{\mathrm{eq}} + x_3, \tag{2}$$

$$\dot{x}_3 = -\alpha x_3 + h x_2,$$

where $x_1 = \ln(GI/\kappa_0)$, $x_2 = G(N_2 - N_1)/\kappa_0$, $x_3 = -P_{\mathrm{eq}} + (G/\kappa_0)\{[(\gamma_1 - \gamma_2)/2\gamma_R](N_1 + N_2) + M_2 - M_1\}$, $P_{\mathrm{eq}} = 0.3887$, $\kappa' = \kappa_0/\gamma_R$, $\Gamma_1 = (\gamma_1 + \gamma_2 + 2z\gamma_R)/2\gamma_R$, $P_0 = \gamma_2 P G/\kappa_0\gamma_R$, $\Omega = 2\pi f$, and the time has been rescaled as $t = \tau/\gamma_R$.

The reduction has been performed observing that, after a suitable change of variables, the full model consists of two blocks: the first one (containing two equations) is nonlinear, while the second (containing the remaining three equations) is linear and presents a frequency response which suitably matches the transfer function of a low-pass first-order filter. Thus, the three equations can be replaced with the third equation of (2) for the variable x_3, while the values of the parameters $\alpha = 0.9667$ and $h = 9.4656$ can be obtained imposing that the two transfer functions have the same amplitude for $\omega = 0$ and for $\omega = 2\pi f_0$, f_0 being the cutoff frequency (the detailed theory is presented in Ref. [10]).

Redefining $P' = P_0 + P_{\mathrm{eq}}$ and $\tilde{\kappa}(\tau) = 1 + m\ \sin\Omega\gamma_R\tau$, and considering Eqs. (2) in the frequency domain ($s = i\omega$), we can represent the system as in Fig. 1(a), where $L(s)$ and $G(s)$ are the linear transfer functions

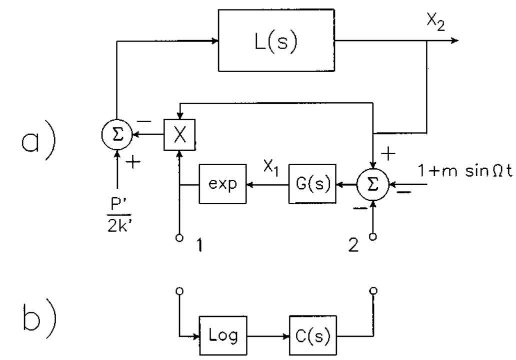

FIG. 1. (a) Logical diagram corresponding to Eq. (2). The blocks $L(s)$ and $G(s)$ correspond to the operators of Eq. (3). (b) Feedback control circuit. $C(s)$ is the filter of Eq. (4), ''Log'' is a logarithmic amplifier. In (a), where the boxes correspond to conceptual operators, 1 and 2 are physically accessible points to which the feedback circuit (b) is connected.

$$L(s) = \frac{2\kappa'(s + \alpha)}{s^2 + s(\Gamma_1 + \alpha) + (\alpha\Gamma_1 - h)},$$
$$G(s) = \frac{\kappa'}{s}. \tag{3}$$

It is interesting to observe that the reduced model (2) presents a structure directly comparable with the standard two dimensional rate equation model. The addition of a filtering process (the third equation) and a constant correction to the pump term (P_{eq}) takes into account the ballast effect induced by the coupling among all the populations. Similarly, in the frequency domain, the system of Fig. 1(a) can be obtained from the standard rate equations, the only difference being the simplified expression of $L(s)$ where α and h are set equal to zero.

The control method here implemented is based upon a feedback loop [Fig. 1(b)] wherein all unwanted frequency components are transmitted as correction signals. The only frequency components which are not affected by the loop are the zero frequency (which controls the long time behavior) and the modulation frequency which, as a consequence, gives rise to a stable periodic orbit. This is achieved by the insertion of a selective filter with a transfer function containing two zeros, at $\omega = 0$ and $\omega = \Omega$, and a maximum at $\omega = \Omega/2$ [11]. This ''intuitive'' structure of the filter (called ''washout filter'') is modeled by the following transfer function:

$$C(s) = \beta\frac{s(s^2 + \Omega^2)}{[s^2 + \xi\Omega s + (\Omega^2/4)](s + \mu\Omega)}, \tag{4}$$

where $\xi = 0.4$, $\mu = 1.5$, and β is the gain factor (see Fig. 4). Driving the filter by the logarithm of the intensity (that is a signal proportional to x_1) and entering into the modulation summing point (note that the filter dephasing essentially goes to zero at $\omega = \Omega/2$), is clearly equivalent in Fig. 1 to modify the integrator $G(s)$ into

$$\tilde{G}(s) = \frac{\kappa'}{s + \kappa' C(s)},$$

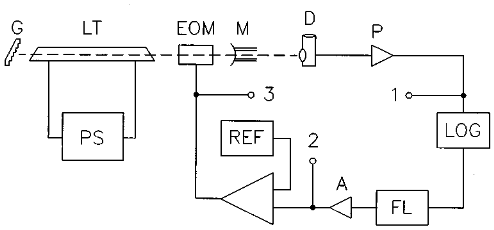

FIG. 2. Experimental setup: G, grating; LT, laser tube; PS, power supply; EOM, electro-optic modulator; M, mirror; REF, reference oscillator providing the sinusoidal driving signal; D, detector; P, preamplifier; LOG, logarithmic converter; FL, washout filter; A, variable gain amplifier. The points 1, 2, and 3 denote available outputs.

so that $\widetilde{G}(0)=G(0)$ and $\widetilde{G}(i\Omega)=G(i\Omega)$, while $|\widetilde{G}|$ has a minimum for $s=i\Omega/2$. This ensures in a simple way that the subharmonic frequency $\Omega/2$ leading to the flip bifurcation can not be present in $x_1(t)$ and a stable period-1 solution can be obtained.

Numerical simulations confirm the validity of our scheme and predict that the feedback signal, with control on, would have an amplitude of the order of 3% of that of the external driving signal (proportional to m).

The experimental setup is shown in Fig. 2. The cavity losses have been modulated by driving the intracavity electro-optic crystal (EOM) with a sinusoidal signal from a reference oscillator (REF). When the modulation frequency is $f=110$ kHz, the period-1 limit cycle appears for $0<m<0.05$, and the first chaotic window (generated after the period doubling cascade) occurs for $0.14<m<0.24$. These two intervals of m correspond to a signal from the reference oscillator with amplitude in the ranges [0, 0.11 V] and [0.32 V, 0.52 V], respectively. In the control loop, the laser intensity is detected, converted with a logarithmic amplifier (with 5 MHz bandwidth and accuracy better than 2%) and filtered so that only the signal with pulsation $\Omega/2$ is fed back to the electro-optic crystal (the numbers 1, 2, and 3 design the outputs where it is possible to measure the feedback signal). Figure 3 presents the electronic scheme of the filter and its frequency response [compared with $C(s)$], respectively.

Figures 4 and 5 show the main results of the experiment. Figure 4(a) reports the chaotic laser oscillations ($m=0.18$, driving signal amplitude 0.4 V) observed in the output point 1 with open control loop, while Fig. 4(b) presents the corresponding feedback signal (measured in the output point 2). The same signals are reported in Figs. 4(c) and 4(d), respectively, but in the case of closed control loop. The control signal of Fig. 4(d) has an amplitude less than 5 mV which, compared with the amplitude of the driving signal (0.4 V), yields a 1.25% perturbation. In order to have a confirmation of the perturbation smallness we have also measured the amplitude of the high voltage signal (output point 3) which drives the electro-optic crystal. We have observed that closed control loop operation induces a relative reduction of the unperturbed signal which is less than 3%. Figure 5 presents a typical transition from chaotic to periodic dynamics (same experimental conditions as in

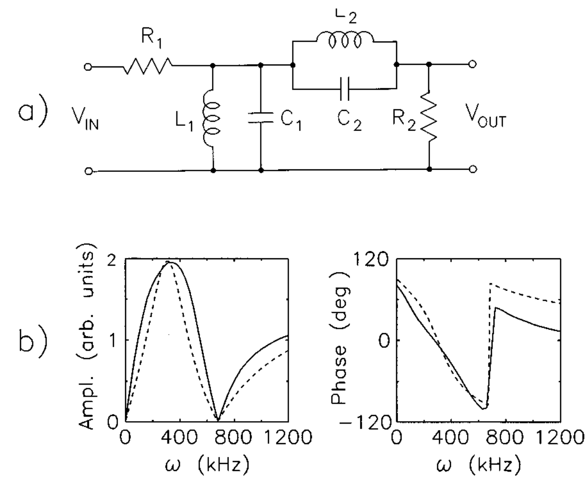

FIG. 3. (a) Electronic scheme of the washout filter: $R_1=1.0$ kΩ, $L_1=6.5$ mH, $C_1=0.1$ nF, $L_2=12.4$ mH, $C_2=0.2$ nF and $R_2=6.7$ kΩ. (b) Amplitude and phase response curves of the washout filter as a function of ω: solid and dashed lines denote the experimental and the theoretical [$C(s)$] filter, respectively.

Fig. 4), characterized by a transient decay towards the final state (the points represents maxima in the laser intensity signal).

Regarding the possibility of stabilizing other period-n orbits with our control scheme, we can observe that the more obvious way is to add other zeros in the filter transfer function (i.e., to stabilize the period-2 orbit a further zero at $\omega=\Omega/2$ is needed). Nevertheless, we have experimentally observed that slight reductions of the gain of the feedback loop (through the variable gain amplifier denoted A in Fig. 2) lead to the stabilization of period-2, period-4, and period-8 limit cycles. However, this results in an increment of the

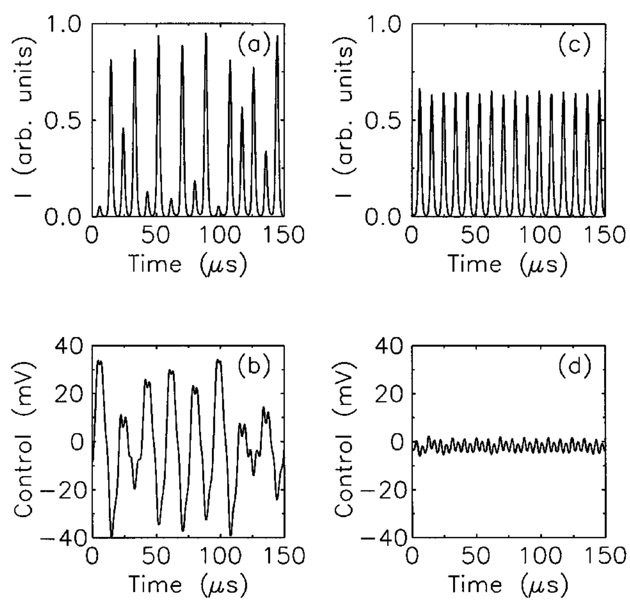

FIG. 4. Experimental results: (a) chaotic laser intensity without control and (b) corresponding control signal; (c) and (d) represent the same signals as (a) and (b), respectively, but in the case of activated control.

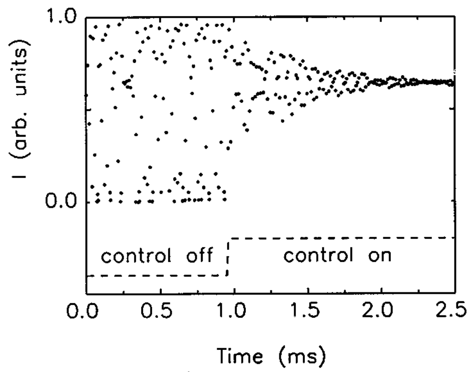

FIG. 5. Typical transition from chaotic to stable period-1 oscillations when the control is activated; the points represent maxima in the laser intensity signal.

relative value of the perturbation with respect to the driving [for example, in the period-8 case, the signal corresponding to Fig. 4(d) reaches an amplitude of about 20 mV], due to the fact that our scheme is originally planned to stabilize the period-1 orbit.

At last, it can be interesting to make some important concluding remarks. Besides the fact that low energy is required, the control scheme presented in this paper can be in principle very fast. As a matter of fact, in our experiment, the feedback loop is entirely realized by analog electronics. Moreover, the control is very robust, since it is independent of laser parameter fluctuations, and the application of this control strategy to a nonautonomous system, such as a modulated laser, is not restrictive. It could also be applied to autonomous systems (laser with electro-optic feedback or with intracavity saturable absorber) which display some dependence of the fundamental oscillation frequency on intrinsic experimental drifts. We have, in fact, verified that the control still works if the driving frequency is varied about $\pm 5\%$.

As a final consideration, we observe that the main point of our treatment is taming of chaotic behavior originating by a nonlinearity of the form $x_2 e^{x_1}$. Since this nonlinearity is the typical one contained in the standard laser rate equations we argue that this control method can be successfully applied to any chaotic laser provided its destabilization occurs through subharmonic bifurcations.

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